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Abstract

This paper studies how Bayesian agents' beliefs about the value of a random variable respond to the disclosure of public information. We show that the release of public information can increase disagreement about the value of that variable. This occurs when the public information does not pertain directly to the value of the variable, but instead pertains to factors influencing its value. This result holds for a range of assumptions about the information structure including cases where the public announcements involve aggregating private information held by the agents.

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1 Introduction

Consider a group of agents who share a common prior about the value of a random variable (e.g., the value of a firm, the quality of an incumbent politician, or the inflation rate) but, after receiving private information, potentially hold different beliefs about it. If these agents are Bayesian, it seems natural that the release of public information should lead to a convergence in their beliefs.[1] If the public information points at, say, a high value of the variable, then all agents should update their beliefs upward, with the magnitude of the update being larger for agents with less optimistic beliefs. This simple intuition is sound if the public information pertains directly to the value of the random variable, e.g., the firm releases information about expected earnings, an NGO releases encompassing information about the quality of an incumbent, or the federal reserve publishes an inflation forecast. But, in many situations, the public information does not pertain to the value of the random value but instead pertains to some of the factors influencing its value, e.g., a decision made by the management of the firm or at shareholders' meeting; a measure of an incumbent's performance along a specific dimension such as the graduation rate of students in public schools or the misuse of public funds; or economic indicators about unemployment or consumer confidence.

In this paper, we show that, in such situations, the release of public information can *increase* disagreement among agents who share a common prior, even if they are Bayesian. Our analysis highlights that the result holds whether or not the factors influencing the value of the variable are correlated and even if the public announcement is the result of aggregating private information held by the agents. As explained below these cases are especially relevant empirically.

To prove these results, we develop a model with Bayesian agents who share a common prior about an unobservable state of the world, and have a common understanding of the information environment. Each agent receives a private signal that is informative about the state of the world and forms beliefs about the value of a random variable based on that information. The actual value of the random variable is determined by two factors: the binary state of the world and a binary action. The value is higher if the state of the world and the action match.

In order to clarify the mechanism and connect to different applications, we consider two different decision-making processes underlying the choice of action: (i) a single decision-maker seeks to match the action and the state based on the information she holds (which

[1] Baliga et al. (2013) and at Benoit and Dubra (2019) provide straightforward regularity conditions under which this is true.

is potentially public); (ii) the agents themselves collectively serve as the decision-maker selecting an action via voting after they observe their private signals. Our analysis covers the cases where the public announcement, which is released after the agents have received their private signals, is either about the action taken or about the state of the world.

Key to our result is that agents who possess different interim beliefs about the state of nature—i.e., beliefs after receiving their private signals but not the public one—may nonetheless hold similar interim beliefs about the value of the random variable.[2] To understand this point, let us first consider the simplest case in which the action is taken by a single decision-maker independently of the information she holds about the state of the world. Agents may then anticipate that she is choosing between the two actions at random (say, 50% chance for each). All agents then agree that the probability that the correct action will be chosen is 50%. Hence, they all have the same interim beliefs about the value of the random variable, independently of their potentially conflicting beliefs regarding the state of nature.

A public announcement revealing the action taken by the decision-maker aligns agents' beliefs on which action has been taken. Yet, it does not lead to a convergence of their beliefs about the likelihood that the *correct* action has been taken. Indeed, the public announcement makes some agents more optimistic about the probability that the decision is correct (i.e., those who received a private signal which matches with the action taken), and others less optimistic (i.e., those who received a private signal which does not match with the action taken). Disagreement among agents about the value of the random variable is thus increasing following the public announcement.

Let us now consider the case of a decision-maker trying to take the action which matches the state of the world. For instance, the manager of a firm, who observes a report about the state of the economy, and then decides whether to invest in a project whose NPV depends on the state of the economy, or a central banker, who observes a report on unemployment rates, and then decides whether to maintain or relax a contractionary monetary policy. Even if the agents anticipate that the decision-maker is now more likely to choose an action that matches the state than not, agents who possess different interim beliefs about the state of nature may still hold similar interim beliefs about the value of the random variable. This is because agents with different information about the state of the world also have different beliefs about which action will be taken. They can thus be similarly optimistic about the fact that the decision-maker will be taking the correct action, even if they have opposite beliefs about which action she will take. A public announcement about the action taken by the decision-maker can then increase disagreement among agents: it reinforces the beliefs of

[2] In the terms of Mongin (2016), we may say that these agents then exhibit spurious unanimity about the value of the random variable.

agents who already thought that the chosen action was likely to be chosen by the decision-maker, but contradicts the beliefs of agents who thought that the other action was likely to be chosen. Their valuations of the random variable have thus diverged. The key is again that, while the public announcement aligns agents' beliefs on which action has been taken, it does not necessarily lead to a convergence of the beliefs about the likelihood that the *correct* action has been taken.

A similar polarizing effect of public information can occur when that information pertains to the state of the world instead of the action taken. This requires that such a public signal is also (indirectly) informative about the action the decision-maker will take, as it is the case when, e.g., the agents anticipate that the decision-maker uses the public signal to make a value-maximizing decision. For example, the release of economic numbers suggesting that the country is about to enter in recession may lead the government to implement stimulus measures to boost the economy. Alternatively, a weather forecast predicting a high likelihood of heavy rain in the coming months can affect agricultural firms' production decisions (e.g., in terms of irrigation, timing of harvest, and type of crops).

Finally, we turn to a variant in which the public announcement only reveals information that agents hold (i.e., their private signals). To tie into several of the motivating examples, we focus here on the case in which the action is determined by the agents through voting. For example, in a proxy contest, shareholders decide whether to approve candidates proposed by hedge fund activists to replace incumbent directors. The public announcement is then the outcome of the vote, which may reveal some of the private information held by the agents. We show that unless the outcome of the vote fully reveals all the private information (i.e., all agents vote perfectly in line with their private signal), disagreement about the value of the random variable can also increase after the public announcement in this case.[3] The imperfect revelation of information through the vote implies that agents draw different inferences from the outcome of the vote about the state of the world and, hence, about the probability that the correct decision has been made. This is because some of them know that they have not revealed their signal through their votes. When the outcome of the vote is close, such post-vote private information can have a substantial impact on voters' beliefs, and then lead to a divergence in beliefs about the value of the random variable.

[3] In equilibrium of the voting game, such an imperfect revelation of the private information occurs as soon as the private signals have asymmetric precision. See, for example, Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997, 1998) for early work on the emergence of imperfect information aggregation in common values voting problems when the voting rule is not well-calibrated to the informational environment.

Literature

Our results help explain the well-established fact that, in real-life, public announcements or the provision of the same information to a group of agents often increase disagreement among those agents. For instance, various studies have found that public announcements about a firm's past earnings or decisions made at shareholders' meetings significantly increase investors' disagreement about firm value (and hence lead to a surge in trading volumes on the stock markets).[4] Similarly, there is evidence that providing information to voters about some dimensions of incumbents' performances may increase disagreement among voters about whether the incumbent or the challenger is best, the so-called "backlash" effect (see, e.g., Baysan 2022 and Enriquez et al. 2024).[5]

Our paper is not the first to propose a theory that can explain the fact that disagreement often increases after public announcements. Most of the proposed alternative theories introduce heterogeneity among agents (see, e.g., Kandel and Pearson 1995, Dixit and Weibull 2007, Acemoglu et al. 2009, Kondor 2012, Banerjee and Green 2015, Cespa and Vives 2015, Mailath and Samuelson 2019, and Gentzkow et al. 2023), or non-Bayesian updating caused by behavioral frictions (see, e.g., Odean 1998, Rabin and Schrag 1999, Scheinkman and Xiong 2003, Zimper and Ludwig 2009, and Baliga et al. 2013) in order to generate increases in disagreement after public announcements. Much fewer of those alternative theories consider Bayesian and homogeneous agents (who share a common prior about both the state of the world and the distribution of the signals), as we do.[6] One exception is Andreoni and Mylovanov (2012),[7] which shows that when individuals have private information about a specific aspect of reality (or, in the language of our model, a specific factor influencing the value of the random variable), the revelation of a public signal about a different, and independently determined, aspect of reality can generate disagreement. Our analysis extends that result and its range of applications in two directions. First, we allow for a correlation between the factors affecting the value of the random variable. Such a correlation arises naturally when the value of the random variable is affected by an action taken by a decision-maker, as it is frequently the case in settings of applied interest (see examples above). In this general

[4] See, e.g., Kandel and Pearson 1995, Bollerslev et al. 2018, Li et al. 2022, and Ben-Rephael et al. 2022.

[5] There is also a large literature in psychology providing evidence of the increase in disagreement among agents who are provided the same piece of new information. See references in Andreoni and Mylovanov (2012) and in Guess and Coppock (2020).

[6] Anderson and Holt (1997) and Hill (2017) provide empirical evidence that agents "learn as cautious Bayesians [...]" about both political and non-political facts. They find some bias in the sense that the amount of learning varies with prior beliefs in a non-Bayesian way. But, the magnitude of that bias is small. This highlights the relevance of providing a theory that embraces Bayesian updating.

[7] Other exceptions are Benoît and Dubra (2019), Loh and Phelan (2019), and Bowen et al. (2023), but the models and the mechanisms they identify are further away from ours.

setting, we show that disagreement can increase when both the public and private signals provide direct information about some factors and indirect information about other ones. Second, we allow for the public signal to simply be an imperfect aggregation of the private information that agents hold. This is relevant for situations in which, e.g., the public announcement relates to the outcome of a vote, as is the case with shareholders' meetings.

2 Model

There are n (odd) agents. The set of agents is denoted by N. At any point in time, agent $i \in N$ estimates the distribution of a binary random variable x, using all the information available to her. The value of the variable x depends on the state of the world $\omega \in \Omega = \{\alpha, \beta\}$ and on a decision $k \in K = \{A, B\}$. We assume that:

$$\begin{aligned} x(A, \alpha) &= x(B, \beta) = 1, \\ x(B, \alpha) &= x(A, \beta) = 0. \end{aligned} \tag{1}$$

The model has four stages:

First stage ($t = 1$). Nature draws the state of the world from Ω with equal probability.

Second stage ($t = 2$). Each agent $i \in N$ receives an independent signal $s_i \in S = \{s_a, s_b\}$, drawn in the following manner: if the state is α then s_a is drawn with probability q_α, and s_b is drawn with probability $1 - q_\alpha$; while, if the state is β then s_a is drawn with probability q_β, and s_b is drawn with probability $1 - q_\beta$, with $q_\alpha > q_\beta$.

Third stage ($t = 3$). An additional signal $\hat{s} \in \hat{S}$ is generated, where $\hat{S} \subset \mathbb{R} \backslash \{0\}$ is a finite set. This signal $\hat{s}$ can be interpreted as a measure of the confidence that the state is α and will affect the decision $k \in K$ in the next stage. This signal may be observed by all the agents, or none.

We consider two possible kinds of the additional signal, depending on whether it is directly or indirectly informative about the state of the world.

- *Directly informative.* The signal $\hat{s}$ is drawn independently from the private signals $(s_i)_{i \in N}$, but conditional on the state ω. That is, the additional signal is directly informative about the state of the world, and, when it is publicly observable, adds to the information that agents cumulatively have.

  This is true, for instance, when an independent entity issues a report describing its predictions regarding the potential of an economy, or the profit prospects of a firm.

In that case, the additional signal is informative about the state of the world without relying on the dispersed information that agents already hold.

For simplicity, we focus on the following representative case of such a signal: $\hat{s} \in \{-1, 1\}$, and if the state is α (resp. β), then 1 is drawn with probability $Q > 1/2$ (resp. $1 - Q$).

- *Indirectly informative.* The signal $\hat{s}$ is drawn conditional on the private signals $(s_i)_{i \in N}$. But conditional on these private signals it is independent of ω. In other words, the additional signal contains (part of) the dispersed private information agents hold about the state of the world and does not add to it. Of course, since the signals $(s_i)_{i \in N}$ correlate with the state ω, the additional signal $\hat{s}$ is still informative about ω, but now merely indirectly so.

  The outcome of a voting procedure among privately informed agents is compatible with this description. In that case, the additional signal can still inform each agent about the state of the world (e.g., if agents' voting behavior correlates with their private signals), but only because it contains information other agents have; not because it adds new pieces of information to the system.

  Similar to the directly informative case, we offer insights by examining the following salient example of such a signal. We assume that each agent $i \in N$ simultaneously casts a vote $v_i \in \{-1, 1\}$, where $v_i = 1$ represents a vote for A, and $v_i = -1$ represents a vote for B.[8] The signal is then $\hat{s} = \sum_{i \in N} v_i \in \hat{S} := \{-n, -n + 2, \ldots, n\}$.

Fourth stage ($t = 4$). A decision $k \in K = \{A, B\}$ is made based on the value of the additional signal $\hat{s}$. There is a $p \geq 1/2$ such that decision is $k = A$ with probability p and $k = B$ with probability $1 - p$ if $\hat{s} > 0$, while it is $k = B$ with probability p and $k = A$ with probability $1 - p$ if $\hat{s} < 0$.[9]

We focus on the effect of the release of public information on agents' disagreement. Public information may be released in both stages 3 and 4, or solely in stage 4. In the former case, the additional signal $\hat{s}$ is publicly released in stage 3, while it is not revealed in the latter case. In both cases, the decision k is publicly observed in stage 4.

We measure disagreement at a given stage by the difference between the most optimistic and the most pessimistic belief about x. That is, if x_i^t represents the beliefs of agent i in stage $t \in \{1, 2, 3, 4, 5\}$, the measure of disagreement in stage t is given by $d^t = \max(x_1^t, ..., x_n^t) -$

[8] Agents cast their vote rationally, i.e., in order to maximize their expected utility.

[9] Note that the exact value of p is known to the agents independently of whether the additional signal is revealed to them or not. Our results are robust to incomplete information about p, as long as agents have common beliefs about it.

$\min(x_1^t, ..., x_n^t)$. We are particularly interested in $t = 2$, $t = 3$, and $t = 4$ because in $t = 1$ the agents' estimates are, trivially, identical.

3 Directly Informative Signal

We assume throughout that at least two agents have received different private signals—otherwise, $d^4 = d^3 = d^2 = 0$—and focus first on the second stage of the model, before any public information is revealed.

Proposition 1. *After receiving their private signal, but before receiving any public information ($t = 2$), agents have the same expectation of the value of x even if they receive different private signals.*

The intuition behind Proposition 1 is the following. After receiving their private signals, agents with different private information disagree on the likelihood of each state of the world, since $\Pr(\omega = \alpha|s_a) \neq \Pr(\omega = \alpha|s_b)$. However, they do not disagree on the expected value of the variable x, because this is determined by the probability that the correct decision is made. And this probability is equal to $R := pQ + (1 - p)(1 - Q)$ in both states. Hence, the private signals play no role in determining expectations regarding the value of x, before any public information is revealed. In the Online Appendix, we provide numerical examples illustrating the result in this proposition as well as those in the propositions below.

As a first benchmark, we focus on the case where $p = 1/2$, that is when the decision k is independent from the additional signal $\hat{s}$. Decision k is then independent from the state ω, and the following result is a direct application of Andreoni and Mylovanov (2012).

Proposition 2. *If $p = 1/2$, agents with different private signals agree at stage 3 about the value of x, but they disagree at stage 4, i.e. $d^4 > d^3 = d^2 = 0$.*

In that case, whether the signal $\hat{s}$ is publicly revealed or not at stage 3, the likelihood of the decision aligning with the actual state of the world is just 50-50. As a result, we find that $d^3 = d^2 = 0$. Then, when the decision is revealed, say $k = A$, agents with private signal s_a are more optimistic than those with private signal s_b that the decision is correct, and thus have a higher expectation of the value of x. It follows that $d^4 > 0$.

In the sequel, we focus on the case where the decision depends on the signal $\hat{s}$, i.e. $p > 1/2$.

3.1 Unobservable Signal

We start by investigating the case where the additional signal $\hat{s}$ is not made public. The only public information is then the decision k, revealed at stage 4.

Proposition 3. *Assume that the signal $\hat{s}$ is not publicly revealed. If $p > \frac{1}{2}$, the revelation of the decision results in agents with different private signals disagreeing about the expected value of x, i.e. $d^4 > d^3 = d^2 = 0$.*

Why does the revelation of the decision generate disagreement between agents? Suppose that agents observe the decision $k = A$. Agents then learn that state α is more likely than expected, as decision $k = A$ is more likely to be taken when the additional signal is $\hat{s} = 1$ (rather than when it is $\hat{s} = -1$), an event which is itself more likely when the state is α (rather than when it is β). This effect makes the assessments that the state is α closer for agents with different private signals, but still distinct. As agents also learn that the decision is $k = A$ (with certainty), their valuations of x thus diverge.

3.2 Observable Signal

The following proposition focuses on the case where the signal $\hat{s}$ is publicly revealed. It describes the evolution of disagreement both after the signal $\hat{s}$ is revealed ($t = 3$) and after the decision is implemented ($t = 4$).

Proposition 4. *Assume that the signal $\hat{s}$ is publicly revealed. If $p > \frac{1}{2}$, the revelation of signal $\hat{s}$ results in agents with different private signals disagreeing about the expected value of x, i.e. $d^3 > d^2 = 0$. If additionally $p < 1$, the decision made at $t = 4$ exacerbates the disagreement further, i.e. $d^4 > d^3$.*

Why does the revelation of signal $\hat{s}$ increase disagreement? Suppose that the signal is $\hat{s} = 1$. Agents then learn that: (i) state α is more likely than expected and (ii) policy A is more likely to be implemented than policy B. While agents still disagree on the state of the world, as $Pr(\alpha|s_a, \hat{s} = 1) > Pr(\alpha|s_b, \hat{s} = 1)$, they agree that the likelihood of A being implemented is equal to p. This implies that an agent who received signal s_a (resp. s_b) is now more optimistic (resp. pessimistic) about the adequacy of the eventual decision. This generates the divergence in beliefs about the value of x between the two types of agents.

Let us now turn our attention to the evolution of disagreement between the third and the fourth stages, i.e., between the revelation of the signal $\hat{s}$ and the decision. If $p < 1$, at the end of stage 3, there is still uncertainty about the policy that will be implemented. This uncertainty brings the expectations of two agents who received opposite signals closer because two agents who received different private signals realize that both the policy they believe is best and the one they believe is worst have a chance to be implemented. The uncertainty about which policy will be implemented then decreases the expected valuation of an agent whose signal coincides with the public signal and increases the one of an agent whose signal does not coincide. When the policy is finally implemented, the policy-related uncertainty is resolved, and hence expectations diverge even more.

3.3 Extension: Asymmetric Precision of the Signal

One might wonder whether our result that disagreement increases after the additional signal $\hat{s}$ is revealed (i.e., $d^3 > d^2$) is an artifact of the assumption that that signal is "correct" with the same probability in either state of the world. Indeed, the finding that $d^2 = 0$ requires this assumption. But the result that $d^3 > d^2$ does not. To see this, assume now that if the state is α (resp. β), then $\hat{s} = 1$ is drawn with probability $Q > \frac{1}{2}$ (resp. $1 - Q + \gamma$), and $\hat{s} = -1$ is drawn with the complimentary probability, with $\gamma \in (Q - 1, Q - \frac{1}{2})$. By continuity of $E[x|s]$ and $E[x|s, \hat{s}]$ in γ it follows that $\lim_{\gamma \to 0} d^2 = 0$ and $\lim_{\gamma \to 0} d^3 > \delta$, for some $\delta > 0$, and hence our main observation remains valid even when the additional signal is asymmetric, provided that the asymmetry (i.e., the value of $|\gamma|$) is not too large. In fact, even if the asymmetry is large, there is at least one realization of the additional signal such that $d^3 > d^2$. In other words, a weaker version of our result (i.e., there is a positive probability that disagreement will increase after the revelation of the additional signal) is true independently of the degree of asymmetry in the private and the additional signals.

4 Indirectly Informative Signal

In this section, we consider the case in which the additional signal $\hat{s} \in \hat{S}$ reveals, through voting, (part of) the information that agents hold. To simplify the analysis, we focus on the case in which the vote is binding, i.e., $p = 1$.[10] This implies that agents' beliefs are the same at any point in time "after the vote" (at stages $t = 3$ and $t = 4$): $d^3 = d^4$.

When the voting process fully reveals the private information dispersed among voters, all agents end up with the same beliefs at stage 3, regardless of the private signal they receive at stage 2. Therefore, we focus on the relevant case in which voting is not fully revealing (sometimes called non-sincere voting).

In our setup, whether fully revealing voting is an equilibrium phenomenon depends on whether the signals differ in their informativeness (this is because the states of nature are

[10] Note that our results hold for $p \in (1/2, 1)$.

equiprobable). Hence, we assume that $q_\alpha < 1 - q_\beta$, so that, at the voting stage, there is a mixed strategy equilibrium in which agents who receive signal s_a vote for A and agents who receive signal s_b mix between voting for A and voting B.[11]

The following proposition focuses on disagreement among agents before the vote:

Proposition 5. *Before the vote ($t = 2$), agents receiving different private signals have different expectations of the value of x (i.e., $d^2 > 0$).*

When the decision is made by voting, the probability of making the correct decision in state α is different than in state β. The reason is that voting involves the aggregation of private information, which has asymmetric informativeness in different states. In particular, since we have assumed $q_\alpha < 1-q_\beta$, the probability of making the correct decision in state α is lower than that in state β. This is simply because agents' private signal is more informative in state β, which helps information aggregation. Thus, before voting, an agent with signal s_b has a higher expectation of the value of x than an agent with signal s_a.

Note that this result is closely related to our discussion of why $d^2 > 0$ in the case with directly informative signal and an asymmetric precision of the signal $\hat{s}$. In that case, we also have that the decision is more likely to be correct in one state than the other, which drives a wedge in the valuations of agents who receive different signals.

The next proposition focuses on disagreement among agents after the vote:

Proposition 6. *After the vote ($t \geq 3$), agents who received different private signals but voted similarly disagree on the expected value of x (i.e., $d^3 = d^4 > 0$).*

Consider two agents who receive different signals but nonetheless vote for the same alternative, A. Given a vote tally $v = \sum_{i=0}^{n} v_i$, these two agents draw exactly the same inference about the number of votes for A other agents cast. This means that, without taking into account their private signals, these two agents have the same post-voting beliefs about the state of the world. Yet, because they received different private signals, they end up with different post-voting beliefs.

Note that agents who receive the same signal but vote differently also disagree about the expected value of x after the vote. For instance, agents who received s_b and voted for B put a higher weight on $\omega = \alpha$ than those who received s_b but voted for A. This is

[11] See that $q_\alpha < 1 - q_\beta$ is a necessary but not sufficient condition for having a mixed strategy equilibrium. If $1 - q_\beta$ is close to q_α, and n is not too large, sincere voting is an equilibrium and therefore full information aggregation is achieved. However, for any $q_\alpha \neq 1 - q_\beta$ there is an $\overline{n} < \infty$ such that for any $n > \overline{n}$ there is a mixed strategy equilibrium. The exact condition for a mixed strategy equilibrium to exist is $\log\left(\frac{q_\alpha}{q_\beta}\right) \geq \frac{n+1}{n-1} \log\left(\frac{1-q_\alpha}{1-q_\beta}\right)$. This is a standard result in the voting literature. See, e.g., Bouton et al. (2018) and references therein.

because, for a given vote tally v, agents who voted for B observe a higher number of votes in favor of A among others' votes. Such agents have intermediate disagreement with respect to those described in the proposition, which is why we ignore them in the result's statement (remember that we focus on the maximal disagreement).

It remains to determine whether agents' disagreement increases or decreases after the vote: that is, is $d^3 = d^4$ larger or smaller than d^2? We can prove the following result when n grows large:

Proposition 7. *If the election outcome is close to a tie (i.e., $v = 1 + 2r$ with $r \in \mathbb{Z}$) then if n is sufficiently large, the disagreement after the vote is larger than the disagreement before the vote (i.e., $\lim_{n\to\infty} d_3 = \lim_{n\to\infty} d_4 > \lim_{n\to\infty} d_2$).*

Proposition 7 proves that when n grows large, if the election is close to a tie, the disagreement among agents is necessarily larger after than before the vote. Note that how large n needs to be depends on r. The intuition of this result is in two parts. First, when n grows large, the disagreement before the vote vanishes. This is because agents expect the outcome of the vote to be correct (A in state α and B in state β) with a probability that tends to 1. Second, when n grows large, agents disagree after the vote if the tally is close to a tie. In that case, the outcome of the vote does not provide much information to the agents about the state of nature. The difference in valuation for two agents who voted for A but received different signal is essentially the difference in beliefs about the state of nature given that there are $\frac{n-1}{2} + r$ signals s_a in the population or $\frac{n+1}{2} + r$ signals s_a in the population. This difference is positive even at the limit.

5 Conclusions

Our findings highlight that it is essential to distinguish between disagreement about the value of a random variable and disagreement about the factors influencing its value. Agents who disagree about the factors may still agree about the value. In our model, agents may disagree both about which action is most likely to be value-maximizing and about which action will be taken, but still agree about the likelihood that the *correct* action is taken, and hence about the (expected) value of the random variable. Then, public announcements about the factors influencing the value of the variable can increase disagreement among Bayesian agents. This result helps explain the well-established fact that, in real-life, public announcements or the provision of the same information to a group of agents often increase disagreement among those agents (see references in the literature review section above).

The central insights of this paper are relevant for various environments in which the value of the variable of interest depends on multiple factors and the information agents receive pertains to only some of these factors. A leading application is the disagreement among investors about the value of a firm (e.g., Bouton et al. (2022)). But there are other relevant applications, e.g., the effect of economic news or monetary policy decisions on inflation anticipations, the effect of fiscal policy decisions on the market's confidence about the economy, and the effect of government actions on the degree of concern people have about the consequences of global warming.

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A Proofs

Proof of Proposition 1. See that the expected value of x in stage 2 is given by:

$$E[x|s] = \Pr(\omega = \alpha|s) \times Pr(k = A|\omega = \alpha) + \Pr(\omega = \beta|s) \times Pr(k = B|\omega = \beta).$$

Note that

$$\begin{aligned} Pr(k = A|\omega = \alpha) &= Pr(k = A|\hat{s}_a)Pr(\hat{s}_a|\omega = \alpha) + Pr(k = A|\hat{s}_b)Pr(\hat{s}_b|\omega = \alpha) \\ &= pQ + (1-p)(1-Q) =: R, \end{aligned}$$

and

$$\begin{aligned} Pr(k = B|\omega = \beta) &= Pr(k = B|\hat{s}_a)Pr(\hat{s}_a|\omega = \beta) + Pr(k = B|\hat{s}_b)Pr(\hat{s}_b|\omega = \beta) \\ &= (1-p)(1-Q) + pQ = R. \end{aligned}$$

Thus,

$$\forall s \in S, \quad E[x|s] = R \times \Big(\Pr(\omega = \alpha|s) + \Pr(\omega = \beta|s) \Big) = R.$$

Therefore, $d^2 = 0$. □

Proof of Proposition 3. Suppose, without loss of generality, that the decision is $k = A$. We have:

$$\begin{aligned} E[x|s_a, k = A] = Pr(\omega = \alpha \mid s_a, k = A) &= \frac{Pr(\omega = \alpha, s_a, k = A)}{Pr(\omega = \alpha, s_a, k = A) + Pr(\omega = \beta, s_a, k = A)} \\ &= \frac{q_\alpha R}{q_\alpha R + q_\beta (1-R)}, \end{aligned}$$

and

$$\begin{aligned} E[x|s_b, k = A] = Pr(\omega = \alpha \mid s_b, k = A) &= \frac{Pr(\omega = \alpha, s_b, k = A)}{Pr(\omega = \alpha, s_b, k = A) + Pr(\omega = \beta, s_b, k = A)} \\ &= \frac{(1-q_\alpha)R}{(1-q_\alpha)R + (1-q_\beta)(1-R)}. \end{aligned}$$

We conclude by observing that, since $q_\alpha > q_\beta$, we have $E[x|s_a, k = A] > E[x|s_b, k = A]$. Thus, $d^4 > 0$. □

Proof of Proposition 4. Suppose, without loss of generality, that the realization of the additional signal is $\hat{s} = 1$. We have:

$$\begin{aligned} E[x|s_a, \hat{s} = 1] &= Pr(k = \omega|s_a, \hat{s} = 1) \\ &= Pr(\omega = \alpha|s_a, \hat{s} = 1)Pr(k = A \mid \hat{s} = 1) + Pr(\omega = \beta|s_a, \hat{s} = 1)Pr(k = B \mid \hat{s} = 1) \\ &= pPr(\omega = \alpha|s_a, \hat{s} = 1) + (1-p)(1 - Pr(\omega = \alpha|s_a, \hat{s} = 1)) \\ &= (1-p) + (2p-1)Pr(\omega = \alpha|s_a, \hat{s} = 1). \end{aligned}$$

A similar formula applies to $E[x|s_b, \hat{s} = 1]$. Hence, the disagreement between agents who received different signals is:

$$d^3 = (2p-1)\Big|Pr(\omega = \alpha|s_a, \hat{s} = 1) - Pr(\omega = \alpha|s_b, \hat{s} = 1)\Big| = (2p-1)d^4,$$

where d^4 measures the disagreement at time $t = 4$ when the decision is $k = A$:

$$d^4 = |Pr(\omega = \alpha|s_a, \hat{s} = 1) - Pr(\omega = \alpha|s_b, \hat{s} = 1)|.$$

Note that d^4 also coincides with the disagreement at time $t = 4$ when the decision is $k = B$ since $Pr(\omega = \alpha|\cdot) = 1 - Pr(\omega = \beta|\cdot)$. We may write:

$$d^4 = \left| \frac{q_\alpha Q}{q_\alpha Q + q_\beta(1-Q)} - \frac{(1-q_\alpha)Q}{(1-q_\alpha)Q + (1-q_\beta)(1-Q)} \right|.$$

As $q_\alpha > q_\beta$, we obtain $d^4 \geq d^3 > d^2 = 0$. Moreover, if $p < 1$, then $d^4 > d^3$. □

For the remaining proofs, it is useful to introduce two new pieces of notation. The quantity $v' = \sum_{j \in N - \{i\}} v_j$ denotes the number of agents except agent i herself who voted for A. We denote by $m = Pr(v_i = -1 \mid s_b)$ the share of agents who vote for B among those who receive a private signal s_b in the mixed equilibrium.

Proof of Proposition 5. At stage 2, the expectation of firm value of a agent who received signal s_a is

$$\begin{aligned} &Pr(k = \omega|s_a, v_i = 1) \\ &= Pr(k = A|\omega = \alpha, v_i = 1)Pr(\omega = \alpha|s_a) + Pr(k = B|\omega = \beta, v_i = 1)Pr(\omega = \beta|s_a) \qquad (2) \\ &= Pr(v' \geq 0|\omega = \alpha)\tfrac{q_\alpha}{q_\alpha + q_\beta} + Pr(v' < 0|\omega = \beta)\tfrac{q_\beta}{q_\alpha + q_\beta} \end{aligned}$$

Since voters who receive s_b are indifferent between voting for A and B in equilibrium, we can simply consider those that vote for A. Therefore,

$$
\begin{aligned}
&Pr(k=\omega|s_b,v_i=1)\\
&=Pr(k=A|\omega=\alpha,v_i=1)Pr(\omega=\alpha|s_b)+Pr(k=B|\omega=\beta,v_i=1)Pr(\omega=\beta|s_b)\\
&=Pr(v'\geq 0|\omega=\alpha)\tfrac{1-q_\alpha}{2-q_\alpha-q_\beta}+Pr(v'<0|\omega=\beta)\tfrac{1-q_\beta}{2-q_\beta-q_\alpha}
\end{aligned}
\tag{3}
$$

We observe that $Pr(k=\omega|s_a,v_i=1)$ and $Pr(k=\omega|s_b,v_i=1)$ are both convex combinations of the quantities $Pr(v'\geq 0|\omega=\alpha)$ and $Pr(v'<0|\omega=\beta)$, although with distinct coefficients, as $q_\alpha\neq q_\beta$. Given the equilibrium conditions, it is known that the error probability is higher in state α than in state β (formally, this directly derives from (9) below). It follows that $Pr(v'\geq 0|\omega=\alpha)\neq Pr(v'<0|\omega=\beta)$ and thus $d^2>0$. $\square$

Proof of Proposition 6. Throughout the proof, we assume that $k=A$ is the outcome of the voting process. An s_a-agent's expectation of firm value is

$$
\begin{aligned}
E[x|v',s_a,k=A]&=Pr(\omega=\alpha|v',s_a)\\
&=\frac{Pr(v',s_a|\omega=\alpha)}{Pr(v',s_a|\omega=\alpha)+Pr(v',s_a|\omega=\beta)}\\
&=\frac{\binom{n-1}{\frac{n+v'}{2}}z^{\frac{n+v'}{2}}(1-z)^{\frac{n-v'}{2}-1}q_\alpha}{\binom{n-1}{\frac{n+v'}{2}}z^{\frac{n+v'}{2}}(1-z)^{\frac{n-v'}{2}-1}q_\alpha+\binom{n-1}{\frac{n+v'}{2}}y^{\frac{n+v'}{2}}(1-y)^{\frac{n-v'}{2}-1}q_\beta}\\
&=\frac{1}{1+\left(\frac{y}{z}\right)^{\frac{n+v'}{2}}\left(\frac{1-y}{1-z}\right)^{\frac{n-v'}{2}-1}\frac{q_\beta}{q_\alpha}},
\end{aligned}
\tag{4}
$$

where

$$
\begin{aligned}
z&:=Pr(v_i=1|\omega=\alpha)\\
&=Pr(v_i=1|s_a)Pr(s_a|\omega=\alpha)+Pr(v_i=1|s_b)Pr(s_b|\omega=\alpha)\\
&=q_\alpha+(1-m)(1-q_\alpha)
\end{aligned}
\tag{5}
$$

and

$$
\begin{aligned}
y&:=Pr(v_i=1|\omega=\beta)\\
&=Pr(v_i=1|s_a)Pr(s_a|\omega=\beta)+Pr(v_i=1|s_b)Pr(s_b|\omega=\beta)\\
&=q_\beta+(1-m)(1-q_\beta).
\end{aligned}
\tag{6}
$$

An s_b-agent who votes for A expects x to be

$$
\begin{aligned}
E[x|v', s_b, k = A] &= Pr(\omega = \alpha|v', s_b) \\
&= \frac{Pr(v', s_b|\omega = \alpha)}{Pr(v', s_b|\omega = \alpha) + Pr(v', s_b|\omega = \beta)} \\
&= \frac{\binom{n-1}{\frac{n+v'}{2}} z^{\frac{n+v'}{2}} (1-z)^{\frac{n-v'}{2}-1}(1-q_\alpha)}{\binom{n-1}{\frac{n+v'}{2}} z^{\frac{n+v'}{2}} (1-z)^{\frac{n-v'}{2}-1}(1-q_\alpha) + \binom{n-1}{\frac{n+v'}{2}} y^{\frac{n+v'}{2}} (1-y)^{\frac{n-v'}{2}-1}(1-q_\beta)} \\
&= \frac{1}{1 + \left(\frac{y}{z}\right)^{\frac{n+v'}{2}} \left(\frac{1-y}{1-z}\right)^{\frac{n-v'}{2}-1} \frac{1-q_\beta}{1-q_\alpha}}.
\end{aligned}
\tag{7}
$$

Since we assume that both agents vote for A, we have $v' = v - 1$ for both of them. Since $q_\alpha \neq q_\beta$, we have that $E[x|v', s_a, k = A]$ is different from $E[x|v', s_b, k = A]$. The proof for $k = B$ is similar, and thus omitted.

□

Proof of Proposition 7. We want to prove that for fixed r when $n \to \infty$, the disagreement among voters is larger after than before voting if the outcome of the election is a tie. We will first show that $\lim_{n\to\infty} d^2 = 0$ and then that $\lim_{n\to\infty} d^4 > 0$.

Note first that, the mixed strategy of s_b-agents, m, must be such that $\lim_{n\to\infty} z = \lim_{n\to\infty} 1 - y$ (otherwise, conditional on being pivotal, one state becomes infinitely more likely than the other, see, e.g., Austen-Smith and Banks (1996)). One obtains from (5) and (6) that the following must hold at the limit:

$$
q_\alpha + (1-m)(1-q_\alpha) = 1 - q_\beta - (1-m)(1-q_\beta) \quad \Leftrightarrow \quad (1-m)(2 - q_\alpha - q_\beta) = 1 - q_\alpha - q_\beta,
$$

so that $m = \frac{1}{2-q_\alpha-q_\beta}$ at the limit. It follows that, at the limit, $z = \frac{1-q_\beta}{2-q_\alpha-q_\beta}$, which is strictly higher than $1/2$ since $q_\beta < q_\alpha$. Since $z = Pr(v_i = 1|\omega = \alpha)$, this directly implies that

$$
\lim_{n\to\infty} \Pr(v' \geq 0|\omega = \alpha) = \lim_{n\to\infty} \Pr(v' < 0|\omega = \beta) = 1.
$$

Plugging that in equations (3) and (4), we obtain

$$
\begin{aligned}
\lim_{n\to\infty} \Pr(k = \omega|s_a) &= \lim_{n\to\infty} \left(\Pr(v' \geq 0|\omega = \alpha) \frac{q_\alpha}{q_\alpha + q_\beta} + \Pr(v' < 0|\omega = \beta) \frac{q_\beta}{q_\alpha + q_\beta} \right) \\
&= \frac{q_\alpha}{q_\alpha + q_\beta} + \frac{q_\beta}{q_\alpha + q_\beta} = 1,
\end{aligned}
$$

and

$$\lim_{n\to\infty} \Pr(k=\omega|s_b) = \lim_{n\to\infty} \left(\Pr(v' \geq 0|\omega=\alpha) \frac{1-q_\alpha}{1-q_\alpha+1-q_\beta} + \Pr(v'<0|\omega=\beta) \frac{1-q_\beta}{1-q_\alpha+1-q_\beta} \right)$$
$$= \frac{1-q_\alpha}{1-q_\alpha+1-q_\beta} + \frac{1-q_\beta}{1-q_\alpha+1-q_\beta} = 1.$$

Thus

$$\lim_{n\to\infty} d^2 = \lim_{n\to\infty} |\Pr(k=\omega|s_a) - \Pr(k=\omega|s_b)| = 0,$$

that is, for $n \to \infty$, the disagreement before voting vanishes.

We can now compute $\lim_{n\to\infty} d^4$, the limit disagreement after voting. In the mixed equilibrium, a agent who receives signal s_b is indifferent between voting for A and voting for B. This requires that

$$\Pr(\omega=\alpha|s_b)\Pr(v'=0|\omega=\alpha) = \Pr(\omega=\beta|s_b)\Pr(v'=0|\omega=\beta)$$

or:

$$\frac{\Pr(v'=0|\omega=\beta)}{\Pr(v'=0|\omega=\alpha)} = \frac{\Pr(\omega=\alpha|s_b)}{\Pr(\omega=\beta|s_b)}. \tag{8}$$

We also know that

$$\Pr(\omega=\alpha|s_b) = \frac{1-q_\alpha}{2-q_\alpha-q_\beta} \text{ and } \Pr(\omega=\beta|s_b) = \frac{1-q_\beta}{2-q_\alpha-q_\beta},$$

and that

$$\Pr(v'=0|\omega=\alpha) = \binom{n-1}{\frac{n-1}{2}} (z(1-z))^{\frac{n-1}{2}}, \text{ and}$$
$$\Pr(v'=0|\omega=\beta) = \binom{n-1}{\frac{n-1}{2}} (y(1-y))^{\frac{n-1}{2}}.$$

Plugging these objects in (8), we obtain

$$\left(\frac{y(1-y)}{z(1-z)}\right)^{\frac{n-1}{2}} = \frac{1-q_\alpha}{1-q_\beta}. \tag{9}$$

We now consider two agents who both voted for A but received different signals (when n is large, these two agents almost surely exist), so that $v' = v - 1 = 2r$ for both of them. Assuming that $k = A$ (i.e. $r \geq 0$), we may apply (5) and (6) to compute the expected value of the firm given that $v' = 2r$:

$$E[x|v' = 2r, s_a] = \frac{1}{1 + \left(\frac{y}{z}\right)^{\frac{n-1}{2}+r} \left(\frac{1-y}{1-z}\right)^{\frac{n-1}{2}-r} \frac{q_\beta}{q_\alpha}},$$

and

$$E[x|v' = 2r, s_b] = \frac{1}{1 + \left(\frac{y}{z}\right)^{\frac{n-1}{2}+r} \left(\frac{1-y}{1-z}\right)^{\frac{n-1}{2}-r} \frac{1-q_\beta}{1-q_\alpha}}.$$

Using (9) , these expected valuations boil down to

$$E[x|v' = 2r, s_a] = \frac{1}{1 + \frac{1-q_\alpha}{1-q_\beta} \frac{q_\beta}{q_\alpha} \left(\frac{y(1-z)}{z(1-y)}\right)^r}$$

and

$$E[x|v' = 2r, s_b] = \frac{1}{1 + \left(\frac{y(1-z)}{z(1-y)}\right)^r}.$$

As $z = \frac{1-q_\beta}{2-q_\alpha-q_\beta} = 1 - y$ at the limit, we have

$$\lim_{n\to\infty} \frac{y}{z} = \lim_{n\to\infty} \frac{1-z}{1-y} = \frac{1-q_\alpha}{1-q_\beta}. \tag{10}$$

We thus obtain:

$$E\left[x|v' = 2r, s_a\right] \quad \to_{n\to\infty} \quad \frac{1}{1 + \frac{q_\beta}{q_\alpha} \left(\frac{1-q_\alpha}{1-q_\beta}\right)^{2r+1}} \tag{11}$$

and

$$E\left[x|v' = 2r, s_b\right] \quad \to_{n\to\infty} \quad \frac{1}{1 + \left(\frac{1-q_\alpha}{1-q_\beta}\right)^{2r}}. \tag{12}$$

Note that agents voting for B have no valuable private information since their private signal can be inferred from the vote outcome, their expectation of the firm's value must thus be intermediate between the two expectations considered above. Therefore, we have

$$\begin{aligned}
\lim_{n\to\infty} d^4 &= \lim_{n\to\infty} \left|E\left[x|v' = 2r, s_a\right] - E\left[x|v' = 2r, s_b\right]\right| \\
&= \left| \frac{1}{1 + \frac{q_\beta}{q_\alpha} \left(\frac{1-q_\alpha}{1-q_\beta}\right)^{2r+1}} - \frac{1}{1 + \left(\frac{1-q_\alpha}{1-q_\beta}\right)^{2r}} \right|,
\end{aligned}$$

which is positive since $q_\alpha \neq q_\beta$. Thus, for $v = 1 + 2r$, with $r \geq 0$, we have that $\lim_{n\to\infty} d^4 > \lim_{n\to\infty} d^2$. The proof for $r \leq 0$ is similar and thus omitted. □

B Online Appendix: Numerical Examples

In this appendix, we provide a numerical example to further discuss the intuition of our results. We assume that $q_\alpha = 0.6$, $q_\beta = 0.1$, $Q = 0.75$, $p = 0.9$, and $n = 9$ (with the latter value only relevant when the additional signal is indirectly informative). For the sake of expositional clarity, we reproduce the relevant propositions from the main text.

Directly Informative Signal

Proposition 1. *After receiving their private signal, but before receiving any public information ($t = 2$), agents have the same expectation of the value of x even if they receive different private signals.*

Given the values of the parameters, the agents' beliefs at the end of stage 2 about the state and the decision to be implemented are given by the following table:

	$s = s_a$	$s = s_b$
$\Pr(\omega = \alpha, k = A \mid s)$	0.600	0.215
$\Pr(\omega = \alpha, k = B \mid s)$	0.257	0.092
$\Pr(\omega = \beta, k = A \mid s)$	0.043	0.208
$\Pr(\omega = \beta, k = B \mid s)$	0.100	0.485
$E[x \mid s]$	0.700	0.700

Agents who receive different signals have profound disagreements about the likelihood of different states and decisions. But the expected value of x, which is equal to $\Pr(\omega = \alpha, k = A \mid s) + \Pr(\omega = \beta, k = B \mid s)$, is the same for both types of agents.

Proposition 3. *Assume that the signal $\hat{s}$ is not publicly revealed. If $p > \frac{1}{2}$, the revelation of the decision results in agents with different private signals disagreeing about the expected value of x, i.e. $d^4 > d^3 = d^2 = 0$.*

The next table shows how agents' beliefs about the state change after the decision $k = A$ is revealed:

	$s = s_a$	$s = s_b$
$\Pr(\omega = \alpha \mid s)$	0.857	0.308
$E[x \mid s, A] = \Pr(\omega = \alpha \mid s, A)$	0.933	0.509

After agents learn the decision $k = A$, agents' beliefs converge on the decision (it is A with certainty) but are still distinct about the state of the world, as $Pr(\alpha|s_a, A) = 0.933$ and $Pr(\alpha|s_b, A) = 0.509$, which generates disagreement about the value of x, that is, $d^4 = 0.424$.

Proposition 4. *Assume that the signal $\hat{s}$ is publicly revealed. If $p > \frac{1}{2}$, the revelation of signal $\hat{s}$ results in agents with different private signals disagreeing about the expected value of x, i.e. $d^3 > d^2 = 0$. If additionally $p < 1$, the decision made at $t = 4$ exacerbates the disagreement further, i.e. $d^4 > d^3$.*

The next table shows how agents' beliefs about the state and the policy to be implemented change after the public signal $\hat{s} = 1$ is revealed:

	$s = s_a$	$s = s_b$
$\Pr(\omega = \alpha, k = A \mid s, \hat{s} = 1)$	0.853	0.514
$\Pr(\omega = \alpha, k = B \mid s, \hat{s} = 1)$	0.095	0.057
$\Pr(\omega = \beta, k = A \mid s, \hat{s} = 1)$	0.047	0.386
$\Pr(\omega = \beta, k = B \mid s, \hat{s} = 1)$	0.005	0.043
$E[x \mid s, \hat{s} = 1]$	0.858	0.557

After receiving the public signal $\hat{s} = 1$, while agents still disagree about the state of the world, as $Pr(\alpha|s_a, \hat{s} = 1) = 0.947$ and $Pr(\alpha|s_b, \hat{s} = 1) = 0.571$, they now agree that the likelihood of A being implemented is 0.9 (before the public signal, the beliefs that A would be implemented were 0.643 and 0.423, respectively). This generates the divergence in beliefs about the value of x: $d^3 = 0.301$.

The next table indicates relevant beliefs after the policy is implemented:

	$k = A$		$k = B$	
	$s = s_a$	$s = s_b$	$s = s_a$	$s = s_b$
$\Pr(\omega = \alpha, k = A \mid s, \hat{s} = 1, k)$	0.947	0.571	–	–
$\Pr(\omega = \beta, k = B \mid s, \hat{s} = 1, k)$	–	–	0.053	0.429
$E[x \mid s, \hat{s} = 1, k]$	0.947	0.571	0.053	0.429

When a policy is finally implemented, it reduces the uncertainty about the policy implemented further, and this generates even higher disagreement. See that independently of the final decision, the disagreement about the expected value of x is equal to $d^4 = 0.376 > d^3$.

Indirectly Informative Signal

Proposition 5: *Before the vote ($t = 2$), agents receiving different private signals have different expectations of the value of x (i.e., $d^2 > 0$).*

Given the values of the parameters there is a mixed strategy equilibrium in which agents who received signal s_a vote for A and agents who received signal s_b vote for A (B) with probability 0.156 (0.844). Taking into account their signal and equilibrium play, the agents' beliefs at the end of stage 2 about the state of the world and the policy to be implemented are given by the following table:

	$s = s_a$	$s = s_b$
$\Pr(\omega = \alpha, k = A \mid s)$	0.778	0.279
$\Pr(\omega = \alpha, k = B \mid s)$	0.079	0.028
$\Pr(\omega = \beta, k = A \mid s)$	0.014	0.070
$\Pr(\omega = \beta, k = B \mid s)$	0.128	0.622
$E[x \mid s]$	0.906	0.902

Note that, while the valuations differ, the difference is small (0.004, which compares to a maximum disagreement of 1). As we will see below, this is a feature common to all values of the parameters, with the difference vanishing when n grows large.

Proposition 6: *After the vote ($t \geq 3$), agents who received different private signals but voted similarly disagree on the expected value of x (i.e., $d^3 = d^4 > 0$).*

Recall that, before knowing the tally, the difference in beliefs about x between players with different private information was 0.004. How does knowing the tally affect this difference? If the total number of votes for A is between 1 and 8, the difference in beliefs between a agent who received the private signal s_a and another one who received signal s_b and voted for A **increases** (see column $d_3 = d_4$ in the table below). See that the difference is particularly striking when the election is (close to) a tie ($v = -1$ or $v = 1$).

v	k	$\Pr(\alpha \mid v)$	$E[x \mid s_a, v_i = 1]$	$E[x \mid s_b, v_i = 1]$	$d_3 = d_4$
-9	B	0.001	-	-	-
-7	B	0.004	0.991	0.999	0.008
-5	B	0.025	0.946	0.996	0.049
-3	B	0.139	0.740	0.975	0.235
-1	B	0.500	0.315	0.861	0.546
1	A	0.861	0.931	0.500	0.431
3	A	0.975	0.988	0.861	0.127
5	A	0.996	0.998	0.975	0.023
7	A	0.999	1.000	0.996	0.004
9	A	1.000	1.000	0.999	0.001

This result is the confluence of two effects. First, knowing the tally eliminates the uncertainty on which policy will be implemented: A is implemented if $v > 0$ and B is implemented otherwise. Second, because agents receiving signal s_b use a mixed strategy, there is asymmetric information between s_a and s_b agents. In particular, an s_b agent who has voted for A knows that there has been $v - 1$ votes for A plus one vote for A after a signal of s_b, while an s_a agent knows that there has been $v-1$ votes for A plus one vote for A after a signal of s_a. Thus, the two types of agents infer a different overall number of signals s_a and s_b in the population, with the s_b agents inferring about one more signal s_b. And, as suggested by the first column of the table, for relatively close elections, one extra signal in favor of B has a very substantial impact on the belief about the state of nature.

By contrast, when the vote tally is $v = 9$, i.e., all agents vote for A, then the disagreement among two agents with different signals decreases after the vote. This is because, in that case, the vote tally becomes overwhelmingly informative that the state of nature is α. Even if the two types of agents still infer a different overall number of signals s_a and s_b in the population from the vote tally, they are both almost sure that the state is α and hence that the decision is correct.

Proposition 7. *If the election outcome is close to a tie (i.e., $v = 1+2r$ with $r \in \mathbb{Z}$) then if n is sufficiently large, the disagreement after the vote is larger than the disagreement before the vote (i.e., $\lim_{n\to\infty} d_3 = \lim_{n\to\infty} d_4 > \lim_{n\to\infty} d_2$).*

Suppose now that $n \to \infty$. If the election outcome is close to a tie ($v = 1$), the disagreement after voting is substantial:[12]

$$\lim_{n\to\infty} d^4 = \left| \frac{1}{1+\frac{q_\beta}{q_\alpha}\left(\frac{1-q_\alpha}{1-q_\beta}\right)} - \frac{1}{1+\left(\frac{1-q_\alpha}{1-q_\beta}\right)^0} \right| = 0.43,$$

which compares to a maximum disagreement of 1.

[12] The formula to calculate $\lim_{n\to\infty} d^4$ is derived in the proof of Proposition 7. Here we consider the case with $r = 0$.